Metamodeling for Variable Annuity Valuation: What works and what does not

Xiaochen Jing

University of Illinois at Urbana-Champaign

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AFIR-ERM Finance, Investment & ERM

IACA Consulting Actuaries

IAALS Life & Annuities

PBSS Pensions & Social Security

Background and Literature

- **Variable Annuities (VAs)** are separate account life insurance contracts linked to a list of financial instruments with tax benefits
- 2 trillion USD in net assets (25% of US insurance industry's total assets)
- Payoff contingent on market performance, policyholder mortality, and withdrawal behavior
- VAs are difficult to evaluate
  - No direct replicating securities in the market
  - Path dependency
  - Uncertain policyholder behavior

  $\Rightarrow$ Complex financial derivatives

Key Actuarial Problems for Variable Annuities

- **Valuation and Hedging**
  - Background: Black & Scholes (1973), Harrison & Kreps (1979),...
  - Numerical valuation: Bauer et al. (2008), Chen & Forsyth (2008),...
  - Analytical valuation: Milevsky & Salisbury (2006), Feng & Volkmer (2012),...
  - Hedging: Coleman et al. (2007, 2008),...
- **Policyholder Behavior**
  - American option pricing: Milevsky & Salisbury (2006), Dai et al. (2008), Shah & Bertsimas (2008),...
  - Utility optimization: Horneff et al. (2010, 2015), Steinorth & Mitchell (2015), Moenig (2021),...
- **Portfolio Valuation**
  - Gan (2015), Gan & Lin (2017), Wu et al. (2018), Quan et al. (2021),...

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Preview

- **Research Question:** How well do metamodeling approaches work on real-world VA contracts?
  - *All* papers so far rely on synthetic datasets
  - Gan and Valdez (2017): "...extremely difficult, if not impossible, for researchers to obtain real datasets..."

- Extract contract features and build a data set of VAs with GMABs
  - Implement a flexible MC simulation process for VA valuation

- Test Metamodeling with different sampling and learning components
  - Larger sample size ✓
  - Sophisticated learners ✓
  - Sampling methods ✗

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VA Features and Data Construction

VAs with Guaranteed Minimum Accumulation Benefits (GMABs)

- Investment in portfolio of stocks and bonds
  - Policyholder's choice (subject to restrictions)

- If surviving to maturity, policyholder is guaranteed the *benefit base*
  - For a fee, regardless of investment performance

- One of the simplest riders, yet contain **high heterogeneity**:
  - *Maturity*: fixed, multiple fixed, step-up reset, etc.
  - *Benefit base*: combinations of return-of-premium, roll-up, ratchet, etc.
  - *Fees*: base, rate, specialities (such as being tied to market volatility)
  - *Free withdrawals, surrender charge schedule, impact of withdrawals*, etc.
  - ...

VA Features and Data Construction

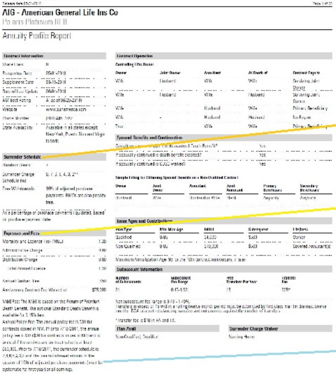
Morningstar Annuity Intelligence

- VA prospectus: Typically several hundred-page long documents with detailed description

- Our source: Morningstar Annuity Intelligence
  - 2,346 VA + GMAB combinations (22,623 in total for all benefit types)
  - Starting from 1994
  - Numerical values on fees and benefits
  - Textual description on features and conditions

VA Features and Data Construction

Morningstar Report Example



Surrender Schedule	
Duration (Years)	7
Surrender Charge Schedule (%)	8, 7, 6, 5, 4, 3, 2**
Free Withdrawals	10% of adjusted purchase payments. RMDs are also penalty free.

As a percentage of purchase payments liquidated, based on purchase payment date.

Expenses and Fees	
Mortality and Expense Risk (M&E)	1.30
Administrative Charge	0.00
Distribution Charge	0.00
Total Annual Expense	1.30
Annual Contract Fee	$50
Anniversary Contract Fee Waived at	$75,000

M&E Fee: The M&E is based on the Return of Premium Death Benefit. The optional Standard Death Benefit is available for 0.15% less.
Annual Policy Fee: The annual policy fee is $30 for contracts issued in NM. Prior to 7/18/2011, the annual policy fee is $35 ($30 for contracts issued in ND) and is waived if the anniversary contract value is at least $50,000. Prior to 7/18/2011, the surrender schedule is 7,6,6,5,4,3,2 and the free withdrawal amount is the greater of 10% of adjusted purchase payments (must be systematic for first year) or all earnings.

Morningstar Report Example

- M&E Fee: The M&E is based on the Return of Premium Death Benefit. The **optional** Standard Death Benefit is available for 0.15% less. Annual Policy Fee: The annual policy fee is $30 for contracts issued in NM. Prior to **7/18/2011**, the annual policy fee is $35 **(**$30 for contracts issued in ND**)** and is waived **if** the anniversary contract value is at least $50,000. Prior to **7/18/2011**, the surrender schedule is 7,6,6,5,4,3,2 and the free withdrawal amount is the greater of 10% of adjusted purchase payments **(**must be systematic for first year**)** **or** all earnings.

- Keywords: optional
- When: Standard Death Benefit
- What: M&E Fee
- How: 0.15% less

VA Features and Data Construction

Example of Feature Extraction

Feature Description	Original Text	Extracted Variable	Variable Value
Benefit Charge	0.750% assessed annually and calculated against the account value	Fee_B_Base Fee_B_Rate	AV (Account Value) 0.0075
Surrender Charge Schedule	8, 7, 6, 5, 4, 3, 2	SC Length Slope Max Min	8, 7, 6, 5, 4, 3, 2 7 -1 8 2
Impact of Withdrawal	Proportionate	IW	Proportionate
Specialty 1	The M&E is based on the Return of Premium Death Benefit. The optional Standard Death Benefit is available for 0.15% less.	Specialty 1	If BenefitOption == Standard Death Benefit: Fee_VA_Rate == Fee_VA_Rate - 0.0015
Specialty 2	(Benefit) Fee percentage is 0.55% prior to 11/1/2010, 0.40% prior to 3/2/2009 and 0.25% prior to 5/1/2008.	Specialty 2	If ID <11/1/2010: fee_B_Rate == 0.0055; If ID <3/2/2009: fee_B_Rate == 0.0040; If ID <5/1/2008: fee_B_Rate == 0.0025;

- 53 contract features for valuation and learning

VA Features and Data Construction

MC Simulation for VA Valuation

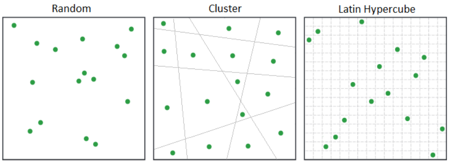
- **Scenario Generation**
  - Investment performance (stock market participation and volatility)
  - Policyholder characteristics (age and sex)
- **Policyholder Behavior**
  - Withdrawal (none, free only, deep out-of-money only, random shock)
  - Step-up (never, always before a certain age, large benefit increase)
- **Financial Model**
  - Standard Black-Scholes model
  - Constant volatility and risk-neutral valuation
- $3 \times 3 \times 3 \times 2 \times 4 \times 3 = 648$ scenarios per contract (1.5 million in total)
  - Evaluation on UW's High Throughput Computing (HTC) Resources

Metamodeling

Sample Selection and Statistical Learning

- **Select** ⇒ **Calculate** ⇒ **Learn (Train and Tune)** ⇒ **Predict**

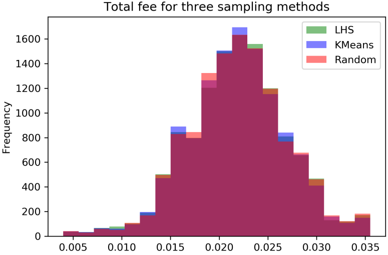
- Representative Sample Selection
  - Random Sampling (benchmark)
  - Clustering (*k*-means)
  - Latin Hypercube Sampling

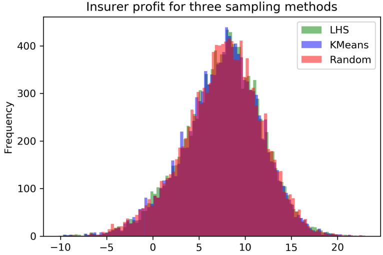


- Statistical Learning
  - Simple GLM (benchmark)
  - Tree-Based Models
  - Neural Network

Representative Observations from the Sampling Component

- Similar distributions on explanatory and response variables





	Random Sample			Latin Hypercube		Cluster Sampling	
Sample Size	1%	5%	20%	1%	5%	1%	5%
Sampling Time (h)	-	-	-	3.42	20.51	3.54	24.74

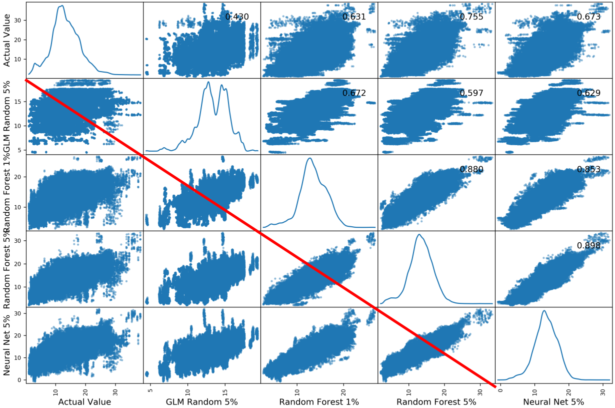
Important Features from the Learning Component

- GLM picks up dummies for feature categories
- Boosted Trees emphasize on fees

GLM		Boosted Trees	
Feature Name	Coefficient	Feature Name	Importance
IW Speciality	10.0609	BenefitFee	11.07%
StepUp Next	8.6718	SubAccountFee L	8.69%
StepUp Initial	-8.3399	VA Fee	6.98%
IW (min)	-5.0211	M&E Fee	6.16%
IW (dollar)	-3.4742	SubAccountFee U	6.06%
FreeWithdrawal Base (AV)	-3.182	WithdrawalStrategy 1	5.12%
FreeWithdrawal Base (BB)	-2.8338	SurranderCharge Slope	4.65%
BenefitFee Speciality	2.6186	Age	3.97%
WithdrawalStrategy 3	-2.4723	AnnuitizationAge	3.87%
BenefitFee Base (max)	2.4433	WithdrawalStrategy 3	3.75%

Results

Correlation of “Actual Values” and Predictions



- Histograms of the “actuarial values” and predictions

Results

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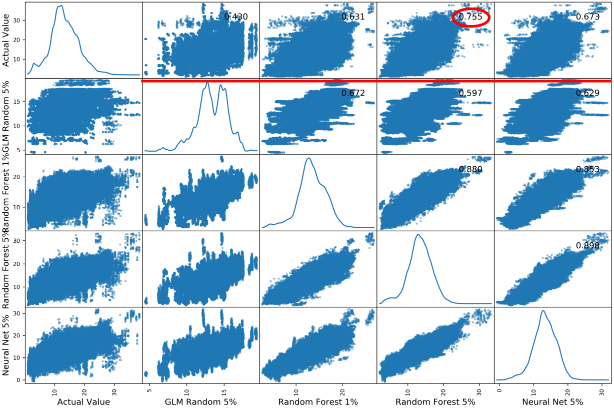

- Scatter plots for correlation

Results

Accuracy and Runtime of Metamodeling

		Random Sample			Latin Hypercube		Cluster Sampling	
	Sample Size	1%	5%	20%	1%	5%	1%	5%
GLM	Tuning Time (h)	-	-	-	-	-	-	-
	OOS RMSE	4.29	4.29	4.27	4.29	4.29	4.30	4.30
Boosted Trees	Tuning Time (h)	1.79	22.17	192.29	2.52	21.43	2.15	21.15
	OOS RMSE	3.77	3.15	3.04	3.78	3.20	3.77	3.21
Random Forest	Tuning Time (h)	0.08	0.43	2.70	0.08	0.44	0.13	0.56
	OOS RMSE	3.69	3.13	2.65	3.70	3.16	3.72	3.14
Neural Network	Tuning Time (h)	7.40	34.15	193.56	7.80	30.50	6.26	25.68
	OOS RMSE	4.00	3.53	3.45	4.02	3.53	4.13	3.55

- GLM isn't improving with more samples

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- Tuning time scales heavily with sample sizes
- RMSE decreases for about 20% with a 20$\times$ increase in sample size

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- RMSE of \$2-\$3, with mean of the actual value around \$13 (s.d. of \$5)
- MAPE of 20%, PE on the portfolio level $< 0.1\%$

Results

Accuracy and Runtime of Metamodeling

- Metamodeling to predict insurer's **profit loading**
  - Similar results for GLM
  - Advanced sampling approaches
    - No effect at 1% sample size, moderate effect at 5% sample size
  - Advanced learning methods
    - Improving prediction at the cost of longer runtime

- Metamodeling to predict VAs from **a single insurer**
  - Significantly reduced runtime and moderately improved accuracy
  - Similar patterns in metamodeling components

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Conclusion

- Constructed a data set of real-world VAs with GMAB riders
  - Extract and formalize payoff related contract features from textual description
- Implemented a flexible simulation based valuation process
  - Accommodate the high number of features and complex structures
- Tested metamodeling with different sampling and learning components
  - Sophisticated learners are better at picking up meaningful relations
  - Larger sample size increases prediction accuracy with longer runtime
  - Sampling methods do not significantly affect the performance

Thank you!